Statics

- *Mechanics* is a branch of physics in which we study the *state of rest or motion of bodies subject to the action of forces*. It can be divided into two logical parts:
  - *statics*, where we investigate the equilibrium of bodies under the influence of forces, and
  - *dynamics*, where we investigate the motion of bodies under the influence of forces.
- Newton's laws of motion

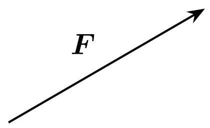
  N1. A particle remains at rest or continues to move in a straight line (with constant velocity) if the resultant force acting upon it is zero.

  N2. The acceleration of a particle is proportional to the resultant force acting upon it and is in the direction of this force.

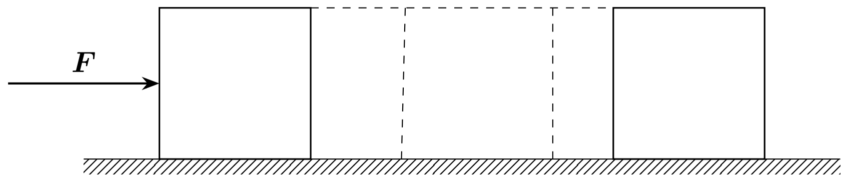
  N3. Two bodies exercise mutual forces upon each other, equal in magnitude but opposed in direction.
- There are four fundamental quantities which occur in mechanics:
  - time (measured in seconds, abbreviation s),
  - distance (measured in metres, abbreviation m),
  - mass (measured in kilograms, abbreviation kg), and
  - force (measured in Newtons, abbreviation N).

Forces

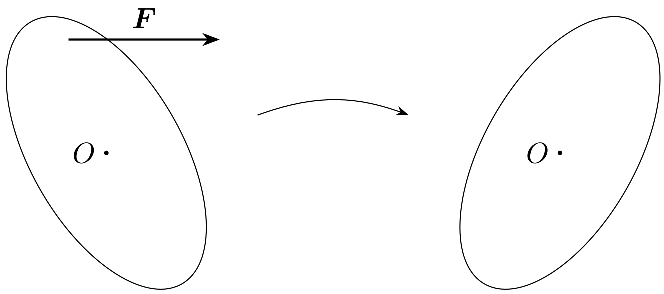
- Force is a vector quantity (experimentally proven).
- Forces can be represented by directed line segments.
  - The length of the line segment gives the magnitude of the force.
  - The direction of the line segment gives the direction of the force.



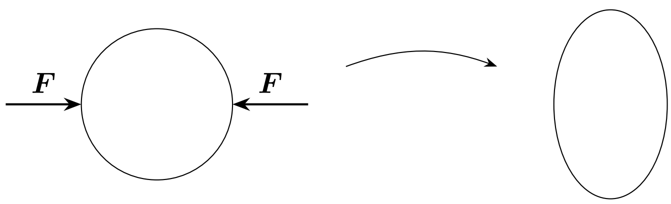
- The *mechanical effect* of a force on a body is what the force does to the body. Examples include:
  - *translation*, when the body moves without any rotation or deformation;



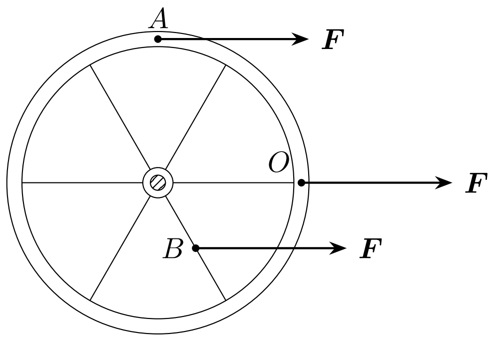
  - *rotation*, when the body changes its orientation about an axis;

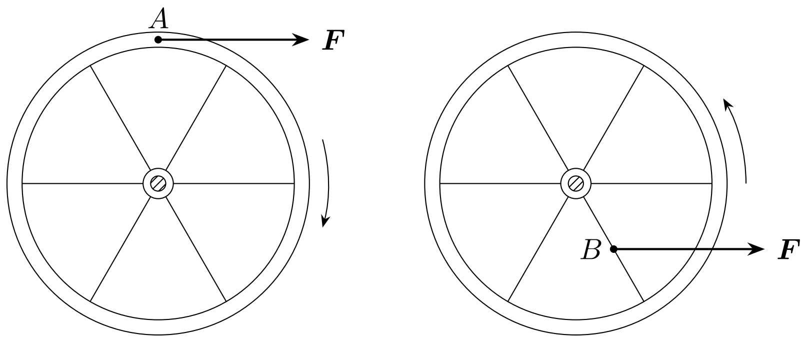


  - *deformation*, when the shape of the body is changed by the forces acting upon it.

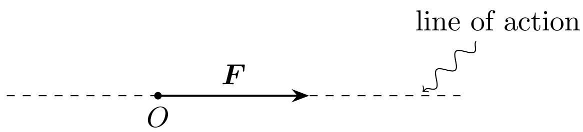


- A *rigid body* is a body of which the deformation is negligible.
- Mechanical effect does not depend only on the magnitude and direction of the force, but also where the force is applied to the body.

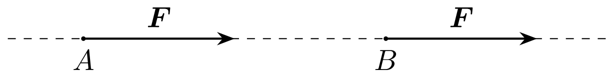


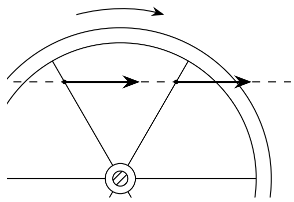


- The point where a force is applied to a body is called the *point of application* of the force. In the figures above, the points O, A, and B are all points of application.
- The straight line through the point of application and parallel to the force is called the *line of action* of the force.

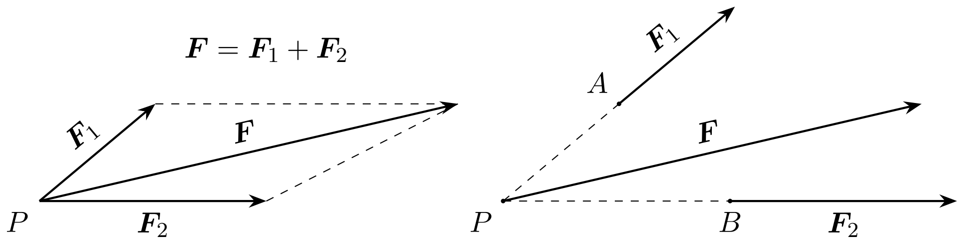


- A force is only fully specified by the provision of its direction, magnitude, and position of its point of application.
- Two forces are *equivalent* (that is, they have the same mechanical effect) if and only if they are equal vectors with the same line of action.



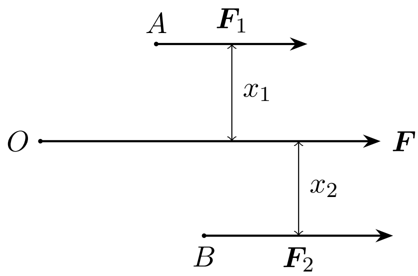


- If two forces $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ are applied at a point of a body, then they can be replaced by an equivalent force which is applied at the same point and is the vector sum of the original two forces.



In the figure above, $\boldsymbol{F}$ is called the *resultant* of the system of forces.

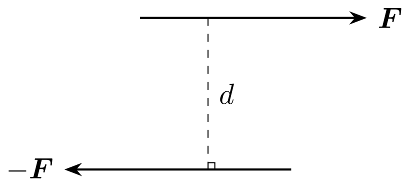
- If two forces $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ are acting upon a body and are parallel to each other, then they can be replaced by an equivalent force which is the vector sum of the original two forces.



Note that the distances x_1 between $\boldsymbol{F}_1$ and $\boldsymbol{F}$, and the distance x_2 between $\boldsymbol{F}_2$ and $\boldsymbol{F}$, satisfy

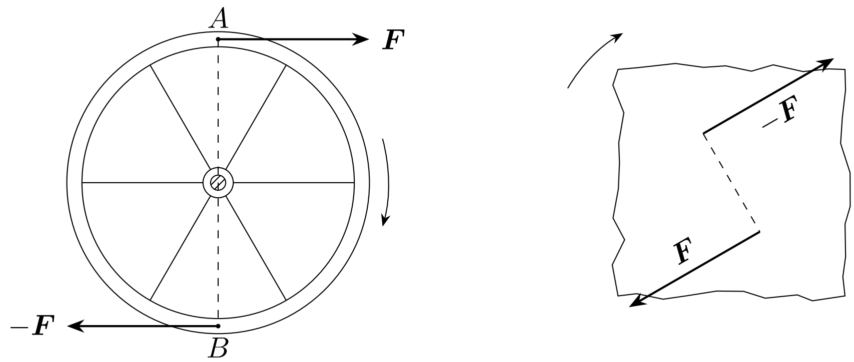
$$\frac{x_1}{x_2} = \frac{F_1}{F_2}.$$

- Two parallel forces, equal in magnitude but opposite in direction and with different lines of action, *cannot be reduced to a single force.*

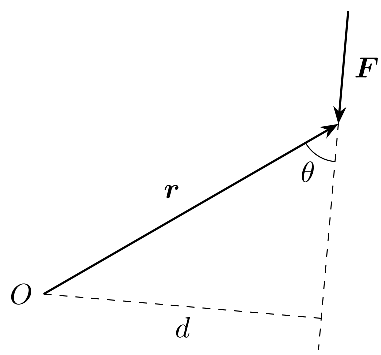


Such a system of forces is called a *couple*, which we indicate by $\{\boldsymbol{F}, -\boldsymbol{F}\}$.

- *A couple can never be reduced to a single force!*
- A couple has a *purely rotational effect* on a body.



- The *moment of a force* is the rotational tendency of a body due to the force acting upon it.



- The moment of a force is a vector quantity. The magnitude of the moment of a force $\boldsymbol{F}$ at the point O is

$$M_O = Fd.$$

It follows from the figure that

$$M_O = Fd = Fr\sin\theta = |\boldsymbol{r} \times \boldsymbol{F}|.$$

Since the vector product appears in the previous equation, we conclude that the moment of a force $\boldsymbol{F}$ is a vector which is perpendicular to the plane containing the vectors $\boldsymbol{r}$ and $\boldsymbol{F}$, and so

$$\boldsymbol{M}_O = \boldsymbol{r} \times \boldsymbol{F}.$$

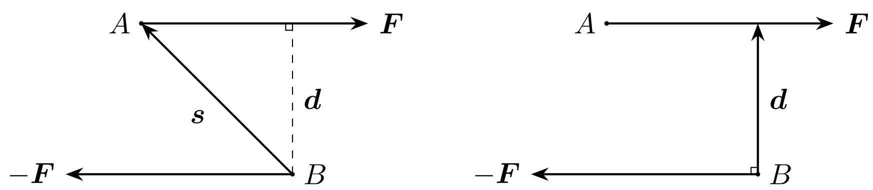
The direction of this vector is given by the right hand rule.

- Equivalent forces have the *same moment about a given point.*
- The resultant rotational effect of a number of forces about a given point is given by the sum of their moments about this point:

$$\begin{aligned}\boldsymbol{M}_O &= \boldsymbol{M}_{O,1} + \boldsymbol{M}_{O,2} + \cdots + \boldsymbol{M}_{O,n} \\ &= \boldsymbol{r}_1 \times \boldsymbol{F}_1 + \boldsymbol{r}_2 \times \boldsymbol{F}_2 + \cdots + \boldsymbol{r}_n \times \boldsymbol{F}_n \\ &= \sum_{i=1}^{n} \boldsymbol{r}_i \times \boldsymbol{F}_i.\end{aligned}$$

- The moment of a couple $\{\boldsymbol{F}, -\boldsymbol{F}\}$ about all points is the same:

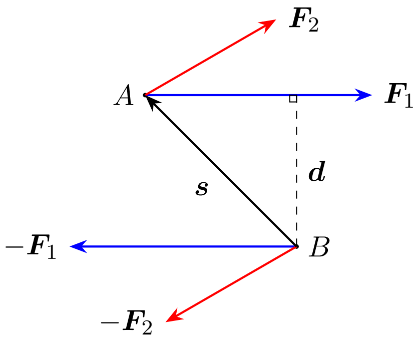
$$\boldsymbol{M} = \boldsymbol{s} \times \boldsymbol{F} = \boldsymbol{d} \times \boldsymbol{F} = Fd\hat{n}.$$



The moment of a couple is a *free vector*, meaning that all couples with the same moment are mechanically equivalent, and contains complete information about the couple's mechanical effect.

- A number of couples are jointly equivalent to a single couple, the moment of which is the vector sum of that of the respective couples.

$$\boldsymbol{M} = \boldsymbol{s} \times \boldsymbol{F}_1 + \boldsymbol{s} \times \boldsymbol{F}_2 = \boldsymbol{M}_1 + \boldsymbol{M}_2$$



- The *resultant of a system of forces* is the most simple system that is mechanically equivalent to it.
- The resultant of a system consists, in general, of:
  - a single force $\boldsymbol{F}$ at an arbitrary point P, which is the vector sum of all the forces in the system, and
  - a couple, where the moment of the couple is the total moment $\boldsymbol{M}_O$ of the given system about P.

  In symbols:
  $$\boldsymbol{F} = \sum_{i=1}^{n} \boldsymbol{F}_i, \quad \boldsymbol{M}_O = \sum_{i=1}^{n} \boldsymbol{s}_i \times \boldsymbol{F}_i.$$
- If $\boldsymbol{F} \cdot \boldsymbol{M}_O = 0$, then the system can be replaced by either an equivalent force or a couple.

Equilibrium of bodies

- A *particle* is a body that can be regarded as small enough in a given situation to be described as a mass at a geometric point.

- A particle is in equilibrium if and only if the vector sum of the forces acting on the particle is the null vector, that is,

$$\sum_{i=1}^{n} \boldsymbol{F}_i = \boldsymbol{0}$$

  or

$$\begin{aligned}
\sum_{i=1}^{n} F_{x,i} &= F_{x,1} + F_{x,2} + \cdots + F_{x,n} = 0,\\
\sum_{i=1}^{n} F_{y,i} &= F_{y,1} + F_{y,2} + \cdots + F_{y,n} = 0,\\
\sum_{i=1}^{n} F_{z,i} &= F_{z,1} + F_{z,2} + \cdots + F_{z,n} = 0
\end{aligned}$$

  in component form.

- This condition of equilibrium can be applied to particles coupled to each other by means of rods, ropes or springs, by applying it to each particle separately.

- A rigid body is in equilibrium if and only if the resultant of the system of forces acting upon the rigid body is the null vector.

- A rigid body is therefore in equilibrium if:

  - the sum of the forces on it is zero, and
  - the total moment of these forces about *any* point is zero.

$$\begin{aligned}
\boldsymbol{F} &= \sum_{i=1}^{n} \boldsymbol{F}_i = \boldsymbol{0}\\
\boldsymbol{M}_P &= \sum_{i=1}^{n} \boldsymbol{s}_i \times \boldsymbol{F}_i = \boldsymbol{0}
\end{aligned}$$

- For a body in a plane we obtain the equations

$$F_x = \sum_{i=1}^{n} F_{x,i} = 0, \quad F_y = \sum_{i=1}^{n} F_{y,i} = 0, \quad M_P = \sum_{i=1}^{n} M_{z,i} = 0.$$