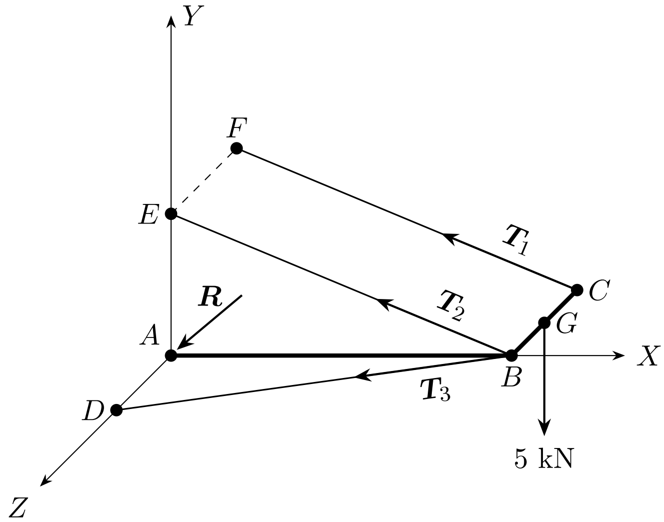
Problem 2.11.20 solution

Consider the following figure illustrating the forces acting upon the beam:



The coordinates of the points A, B, C, D, E, F and G are

$$
\begin{aligned}
A &= A(0,0,0), & B &= B(3,0,0),\\
C &= C\left(3,0,-\frac{3}{2}\right), & D &= D\left(0,0,\frac{5}{4}\right),\\
E &= E\left(0,\frac{5}{4},0\right), & F &= F\left(0,\frac{5}{4},-\frac{3}{2}\right),\\
G &= G\left(3,0,-\frac{3}{4}\right).
\end{aligned}
$$

The components of the forces $\boldsymbol{T}_1$, $\boldsymbol{T}_2$, $\boldsymbol{T}_3$, $\boldsymbol{R}$, and $\boldsymbol{W}$ in the given Cartesian system is

$$\boldsymbol{T}_1 = T_1\hat{n}_1 = T_1\frac{\overline{CF}}{|\overline{CF}|} = T_1\frac{-3\hat{x}+\frac{5}{4}\hat{y}}{\sqrt{9+\frac{25}{16}}} = \frac{T_1}{13}(-12\hat{x}+5\hat{y}) \tag{1}$$

$$\boldsymbol{T}_2 = T_2\hat{n}_2 = T_2\frac{\overline{BE}}{|\overline{BE}|} = T_2\frac{-3\hat{x}+\frac{5}{4}\hat{y}}{\sqrt{9+\frac{25}{16}}} = \frac{T_2}{13}(-12\hat{x}+5\hat{y}) \tag{2}$$

$$\boldsymbol{T}_3 = T_3\hat{n}_3 = T_3\frac{\overline{BD}}{|\overline{BD}|} = T_3\frac{-3\hat{x}+\frac{5}{4}\hat{z}}{\sqrt{9+\frac{25}{16}}} = \frac{T_3}{13}(-12\hat{x}+5\hat{z}) \tag{3}$$

$$\boldsymbol{R} = R_x\hat{x} + R_y\hat{y} + R_z\hat{z} \tag{4}$$

$$\boldsymbol{W} = -W\hat{y} \tag{5}$$

For the system to be in equilibrium, we require (using N1) that

$$\boldsymbol{R} + \boldsymbol{T}_1 + \boldsymbol{T}_2 + \boldsymbol{T}_3 + \boldsymbol{W} = \mathbf{0}. \tag{6}$$

By substituting equations (1), (2), (3), (4) and (5) into equation (6) and suitably factoring and grouping, we obtain the component equations:

$$(x - \text{component}) \qquad R_x - \frac{12}{13}(T_1 + T_2 + T_3) = 0 \tag{7}$$

$$(y - \text{component}) \qquad R_y - \frac{5}{13}(T_1 + T_2) - 5 = 0 \tag{8}$$

$$(z - \text{component}) \qquad R_z - \frac{5}{13}T_3 = 0 \tag{9}$$

The second condition of equilibrium requires that the sum of the moments of the system be the null vector. Consider all the moments about A, then it follows that

$$\begin{aligned}
\sum \boldsymbol{M}_A &= \mathbf{0} \\
\overline{AC} \times \boldsymbol{T}_1 + \overline{AB} \times \boldsymbol{T}_2 + \overline{AB} \times \boldsymbol{T}_3 + \overline{AG} \times \boldsymbol{W} &= \mathbf{0} \\
\overline{AC} \times \boldsymbol{T}_1 + \overline{AB} \times (\boldsymbol{T}_2 + \boldsymbol{T}_3) + \overline{AG} \times \boldsymbol{W} &= \mathbf{0} \\
\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 3 & 0 & -\frac{3}{2} \\ -\frac{12}{13}T_1 & \frac{5}{13}T_1 & 0 \end{vmatrix} + \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 3 & 0 & 0 \\ -\frac{12}{13}(T_2 + T_3) & \frac{5}{13}T_2 & \frac{5}{13}T_3 \end{vmatrix} + \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 3 & 0 & -\frac{3}{4} \\ 0 & -5 & 0 \end{vmatrix} &= \mathbf{0} \\
\left(\frac{15}{26}T_1 - \frac{15}{4}\right)\hat{x} + \left(\frac{36}{26}T_1 - \frac{15}{13}T_3\right)\hat{y} + \left(\frac{15}{13}(T_1 + T_2) - 15\right)\hat{z} &= \mathbf{0}
\end{aligned}$$

Equating the components of the vector on the left-hand side with those of the null vector yields a system of three linear equations in three unknowns:

$$\begin{aligned}
\frac{15}{26}T_1 - \frac{15}{4} &= 0, \\
\frac{36}{26}T_1 - \frac{15}{13}T_3 &= 0, \\
\frac{15}{13}(T_1 + T_2) - 15 &= 0.
\end{aligned}$$

The solution of this system is

$$T_1 = T_2 = \frac{13}{2}, \quad T_3 = \frac{39}{5}.$$

Substituting these values back into equations (7), (8) and (9) and solving for R_x, R_y and R_z yields

$$\begin{aligned} R_x &= \frac{12}{13}(T_1 + T_2 + T_3) = \frac{96}{5} \approx 19,200 \text{ kN}, \\ R_y &= 5 - \frac{5}{13}(T_1 + T_2) = 0 \text{ kN}, \\ R_z &= -\frac{5}{13}T_3 = -3 \text{ kN}. \end{aligned}$$

Thus, the reaction force is

$$\boldsymbol{R} = \left(\frac{96}{5}\hat{x} - 3\hat{z}\right) \text{ kN}$$

with magnitude

$$R = \sqrt{\frac{9441}{25}} \approx 19,433 \text{ kN}.$$